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Measuring Accessibility: Methods & Issues

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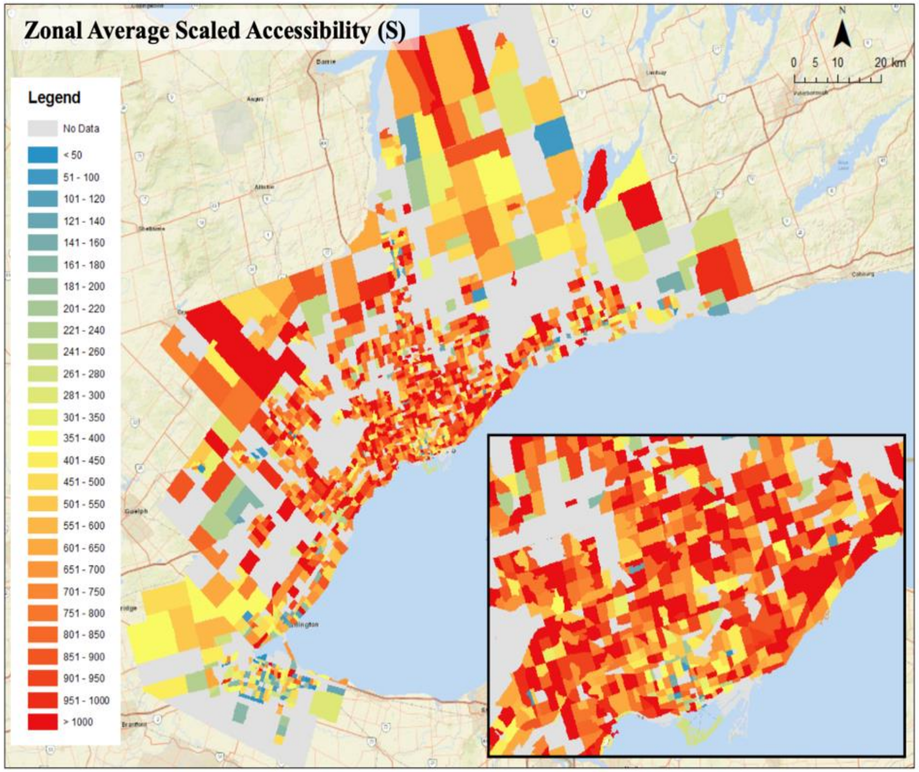
Accessibility is the *potential* for participating in activities (interacting with people & places) that are distributed over space (& time) (Páez, et al., 2012)

This presentation discusses approaches to the quantitative measurement of accessibility.

- Axioms & assumptions.
- A typology of measures.
- Issues in using accessibility measures.

Accessibility Axioms *(Miller, 2018b)*



Accessibility to Employment by Worker Residential Zone, Sales/Service Workers, Greater Toronto-Hamilton Area (GTHA) 2016. Source: Xi (2019)

1. Accessibility is a point measure: it varies from point to point in space.
2. Accessibility is activity (trip purpose) specific.
3. Accessibility intrinsically combines measures of the:
   a) Ease/difficulty in travelling to (interacting with) different points in space; typically referred to as the *disutility*, *generalized cost*, or *impedance* of travel.
   b) *Attractiveness* (desirability) and/or *magnitude* of opportunities at different spatial locations.
4. Specifically, the measurement of accessibility involves the integration (summation) of opportunities over space, weighted by the ease of interaction:

   Opportunities that are closer / easier to access generally are weighted higher than those that are further away / more difficult to reach.

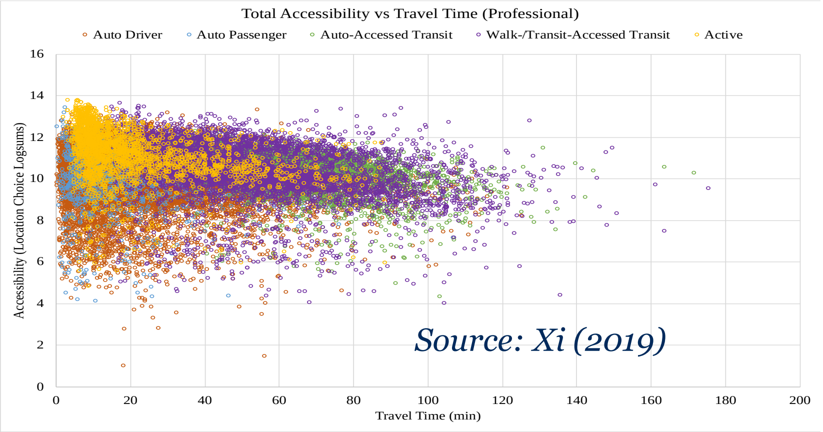
Operational definitions/assumptions are required for …

1. Travel disutility / impedance.
2. Location attractiveness.
3. Role of individual tastes/preferences & constraints in determining travel impedance & location attractiveness.
4. The set of locations to include in the accessibility calculation.

Travel Impedance

- Distance vs. travel time: travel time generally preferred.
- BUT: travel time varies by mode (& time of day).

➡ Time-based accessibilities vary by mode (& time of day).



- Impedance can also be extended beyond travel time to more general measures of the overall *utility* of trip, which can be a function of many attributes (cost, reliability, etc.).

Location Attractiveness

Number of jobs by traffic zone, Greater Toronto-Hamilton Area (GTHA), 2011

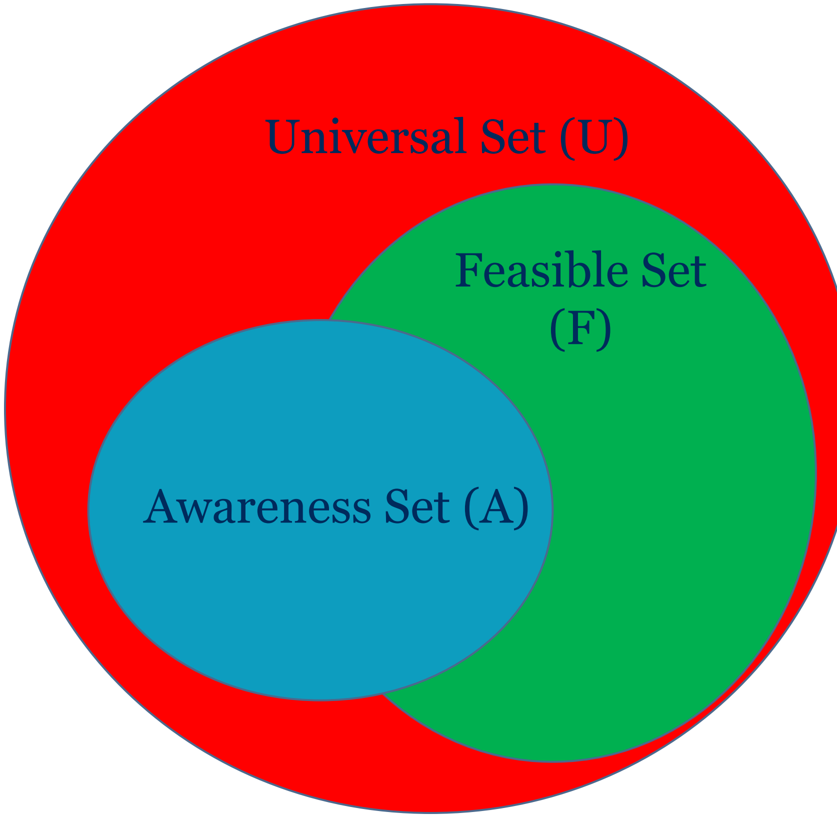
- "Size" is by far the most common variable used to define location attractiveness (number of jobs, number of stores, retail floorspace, etc.).
  - I.e., the "bigger" the location, the more "attractive" it is (the more likely one is to visit it).
  - Assumes all jobs, stores, etc. are the same.
- Could use other attributes of locations, but this is rarely done.

Person-Level Heterogeneity

- Both travel utility and location attractiveness can be expected to vary subjectively from person to person (heterogeneous tastes & preferences).
- Accessibility constraints (access to cars, income constraints, physical mobility constraints, etc.) also vary across individuals.
- Accounting for these differences is generally very important for policy analysis (equity impacts, etc.).
- BUT, this adds additional complexity to accessibility calculations.
- Many accessibility measures are computed at an aggregate zonal level, with little or even no disaggregation by person type.

Location "Choice Set"

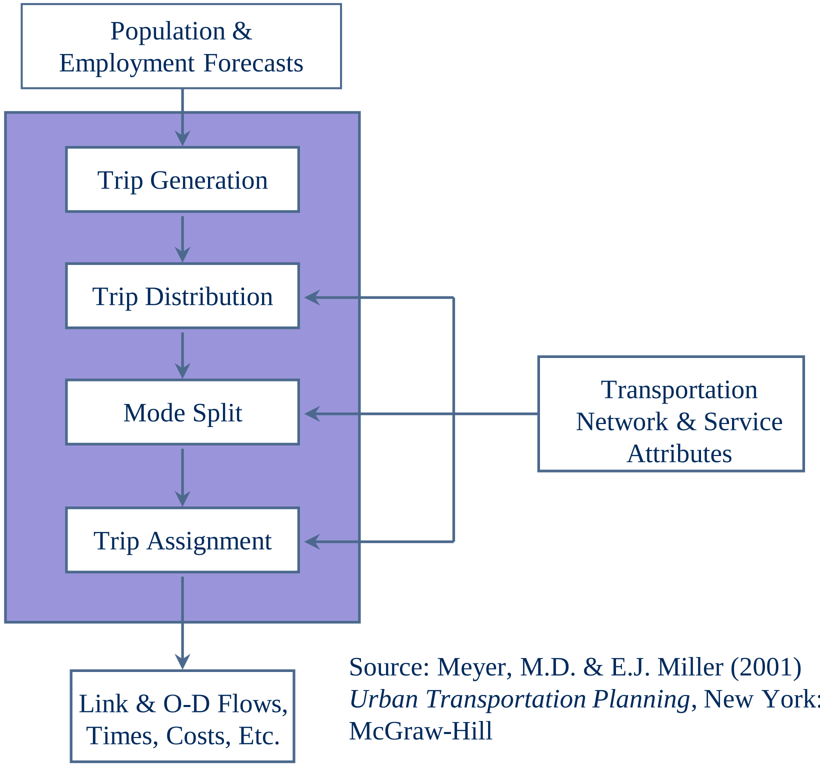
- What locations are relevant in the assessment of accessibility?
- A very challenging technical question.
- Many different assumptions; no strong theory; no universal, standard practice.



Choice set, $C = A \cap F$

Accessibility & Travel Demand

- A logical connection clearly exists between accessibility & travel demand.
- People reveal their preferences for different activity locations & modes of travel through their destination & mode choices.
- Accessibility should, therefore, be consistent with how actual location/travel choices are made.
- ALL operational measures have direct implications for assumptions re. destination/location travel choices & can be associated with explicit models of these choices.



Source: Meyer, M.D. & E.J. Miller (2001) *Urban Transportation Planning*, New York: McGraw-Hill

A Typology of Accessibility Measures

1. Distance/time to the closest location.
2. Cumulative opportunities within an access threshold (isochrone method).
3. Gravity/entropy model denominators (Hanson's measure).
4. Random utility-based measures.

(Handy & Niemeier, 1997; Kwan, 1998)

Distance to Nearest Location

$$A^{ip} = \underset{j \in L^p}{MIN}(d_{ij}) \qquad [1]$$

Where:

A^{ip} = Accessibility of zone i to location of type p

L^p = Set of locations of type p

d_{ij} = Distance (or travel time for a given mode) from i to location j in set L^p

This measure is consistent with an extremely simple location model in which the nearest location is always chosen with probability 1.0. That is:

$$P_j^{ip} = 1 \; if \; d_{ij} = \underset{j' \in L^p}{MIN} d_{ij'} \; ; = \; 0 \; otherwise \qquad [2]$$

Where, P_j^{ip} is the probability of choosing location j for purpose p given that one is located in zone i.

- This measure does not consider the:
  - Size/attractiveness of locations.
  - Cumulative effect of multiple accessible locations.
- Not generally recommended as a stand-alone accessibility measure.

Cumulative Opportunities

$$A^{ip} = \sum_{j \in L^p_{D|i}} X^p_j \qquad [3]$$

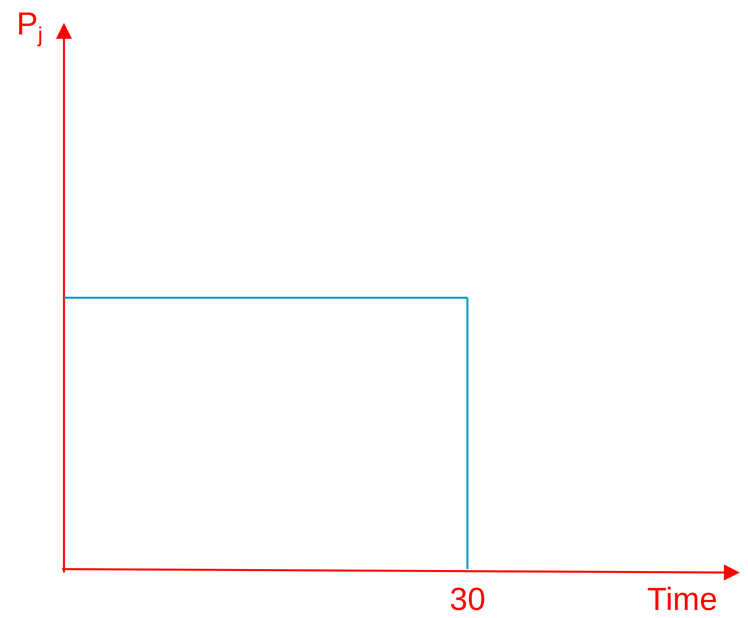
Where:

$L^p_{D|i}$ = Set of locations of type p that are within a maximum distance (or travel time) D of zone i

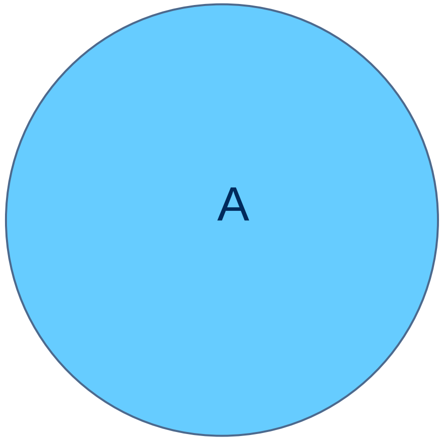
X^p_j = Size of activity type p (number of jobs, stores, etc.) at location j

This accessibility measure is consistent with a location choice model of the form:

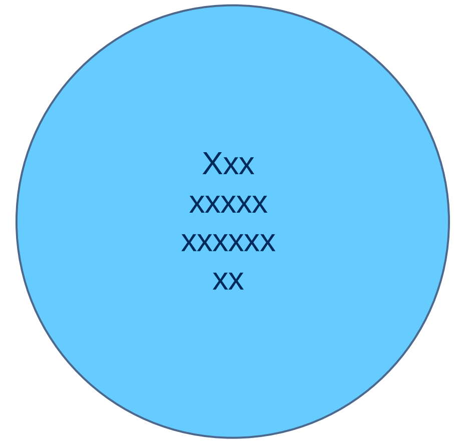
$$P^{ip}_j = \frac{X^p_j}{\sum_{j' \in L^p_{D|i}} X^p_{j'}} \; if\ j \in L^p_{D|i} \; ; = 0\ otherwise \quad [4]$$



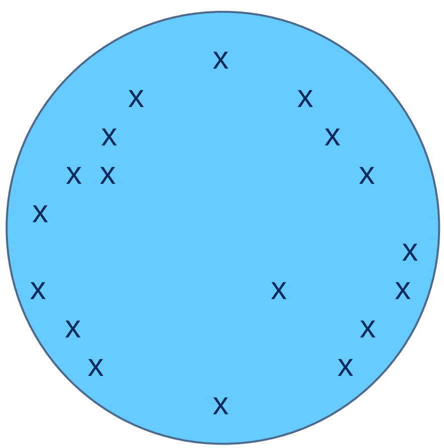
Threshold Effects in Isochrone Accessibility Calculations



Access to jobs from point A:
Same number of jobs, 3 cases



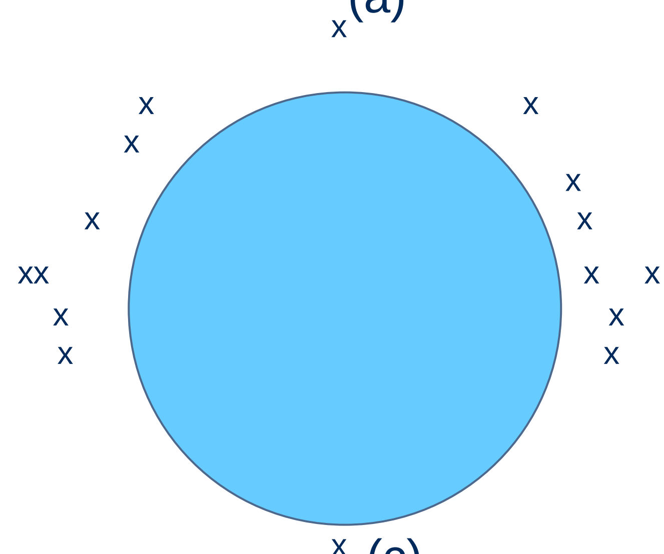
(a)



(b)

$E_a = E_b$

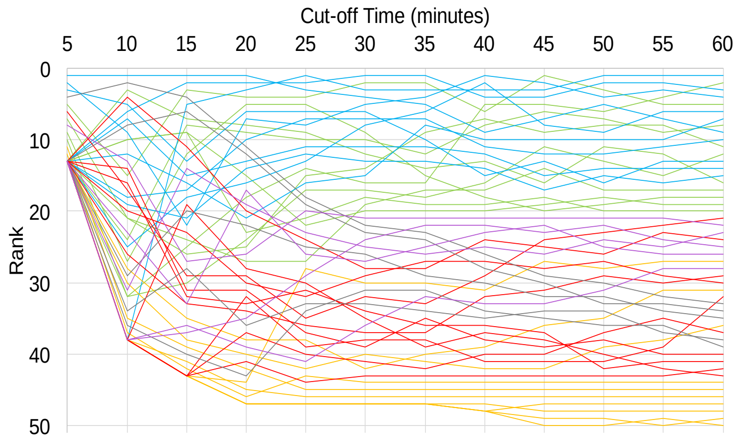
$E_c = 0$



(c)

Other Issues with Isochrone Measures

- Mode-specific (if using travel times); no way to combine across modes.
- Place-based, not person-based.
- No standard threshold.
- Accessibility rankings across zones can vary depending on the threshold used.



Residential zones' accessibility ranks to jobs by threshold (Xie, et al., 2018).

Gravity/Entropy (Hansen, 1959) Measures (1)

$$A^{ip} = \sum_{j \in L^{ip}} X_j^p f(d_{ij}) \qquad [5]$$

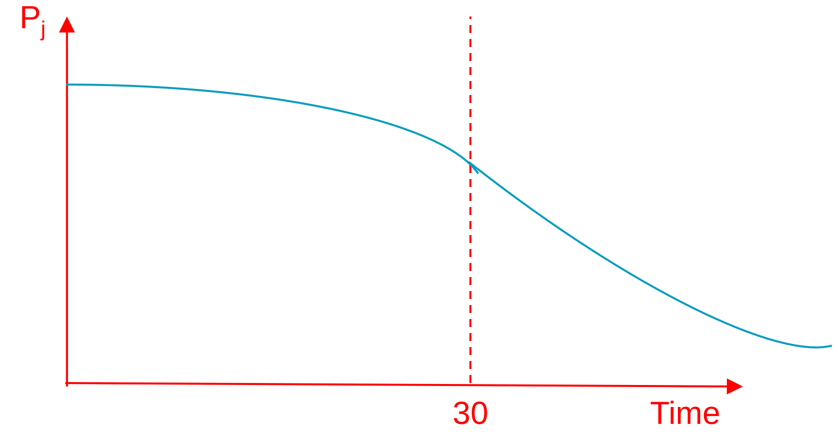
Where:

L^{ip} = Set of locations of type p in the "choice set" for zone i

$f(d_{ij})$ = Impedance function; $\partial f / \partial d_{ij} < 0$

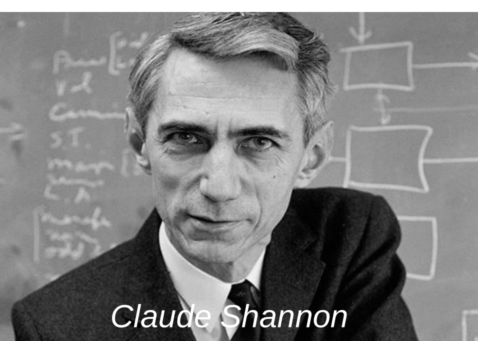
Equation [5] is consistent with a location choice model of the form:

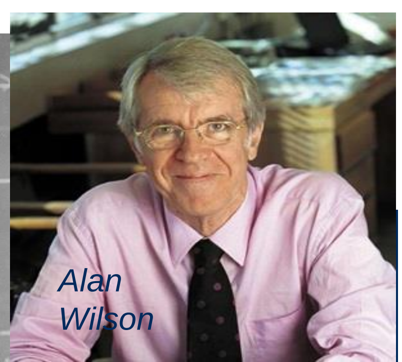
$$P_j^{ip} = \frac{X_j^p f(d_{ij})}{\sum_{j' \in L^{ip}} X_{j'}^p f(d_{ij'})} \qquad [6]$$

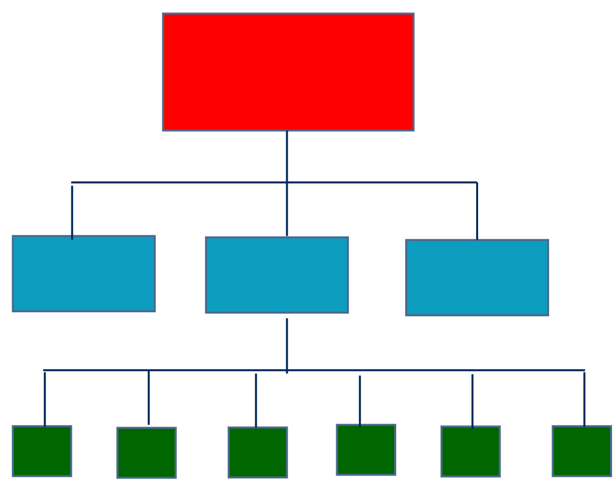


Gravity/Entropy Measures (2)

- Gravity models often appear to be *ad hoc* in derivation, but, in fact, they:
  - Can be derived from Shannon's Information Theory (Shannon, 1948).
  - Are the statistically least-biased (most likely) estimates of spatial interaction, given limited known information about the system (Wilson, 1967; Webber, 1977).







Macro State:
Known information

Meso State:
To be predicted.

Micro State:
Unknown, random

Gravity/Entropy Measures (3)

- Both isochrone & gravity accessibility measures are defined as the *denominator* of their associated destination choice models; i.e.:

Accessibility =

Impedance-weighted sum of total opportunities located within a given set of feasible locations.

OR:

Probability of choosing location j (eqn. [6])=

(Accessibility of zone j) / (Total Accessibility)

Random Utility-Based Measures (1)

Assume that a decision-maker perceives the utility of a destination as:

$$U_j = V_j + \varepsilon_j \qquad [7]$$

where ε_j is the individual's idiosyncratic deviation in terms of how s/he perceives the utility of alternative j relative to the population average utility, V_j. The person chooses the alternative that generates the maximum perceived utility, U_j.

Under very common assumptions, the probability that j is the maximum utility alternative and so is chosen is given by the *multinomial logit (MNL)* model (Train, 2009):

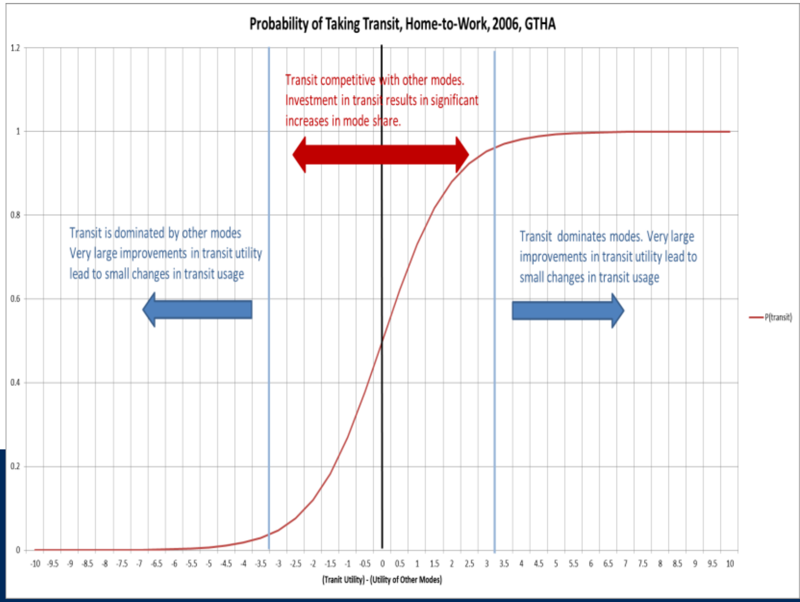
$$P_j^{ip} = \frac{e^{V_j}}{\sum_{j' \in L^{ip}} e^{V_{j'}}} = \frac{e^{\beta Z_j}}{\sum_{j' \in L^{ip}} e^{\beta Z_{j'}}} \qquad [8]$$

Where:

$V_j = \boldsymbol{\beta Z}_j$ = The *systematic utility* of alternative j

$\boldsymbol{Z}_j$ = Vector of explanatory variables

β = (Row) vector of parameters



Random Utility-Based Measures (2)

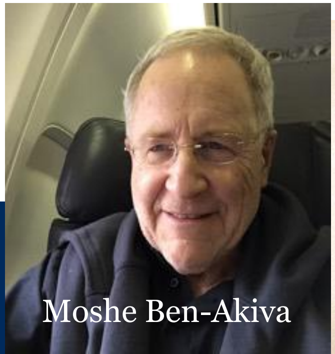
The actual perceived maximum utility is unobservable, but, for the case of the MNL model, it can be shown (Ben-Akiva & Lerman, 1985) that the *expected maximum utility* (I^{ip}) associated with this choice is given by:

$$I^{ip} = E\left[MAX_j\left(U_j\right)\right] = ln\left(\sum_{j \in L^{ip}} e^{\boldsymbol{\beta Z}_j}\right) \qquad [9]$$

That is, it is the natural logarithm of the denominator of the logit choice model (sometimes referred to as the "logsum" term). Further, it can also be shown that this expected maximum utility is the *consumer's surplus* for this choice. Thus it is a standard measure of economic benefit.

Given this, Ben-Akiva and Lerman (1985) argue that it also provides a behaviourally and economically sound definition of accessibility: **accessibility for a given activity is the expected utility that would be derived from participation in this activity, which is also the consumer surplus associated with this participation.** That is:

$$A^{ip} = ln\left(\sum_{j \in L^{ip}} e^{\boldsymbol{\beta Z}_j}\right) \qquad [10]$$


Moshe Ben-Akiva


Daniel McFadden
Nobel Laureate

Gravity & Logit Model Equivalence (1)

Assume a logit destination choice model given by:

$$P_j^{ip} = \frac{e^{ln\left(X_j^p\right) + \gamma d_{ij}}}{\sum_{j\prime \in L^{ip}} e^{ln\left(X_{j\prime}^p\right) + \gamma d_{ij\prime}}} \qquad [11]$$

Now, in the gravity model in equation [6], define the impedance function to be $f\left(d_{ij}\right) = e^{\gamma d_{ij}}$, a very common specification. Equation [6] then becomes:

$$P_j^{ip} = \frac{X_j^p e^{\gamma d_{ij}}}{\sum_{j\prime \in L^{ip}} X_{j\prime}^p e^{\gamma dij\prime}} \qquad [12.1]$$

Noting that $X_j^p = e^{ln\left(X_j^p\right)}$, equation [12.1] can be rewritten as:

$$P_j^{ip} = \frac{e^{ln\left(X_j^p\right) + \gamma d_{ij}}}{\sum_{j\prime \in L^{ip}} e^{ln\left(X_{j\prime}^p\right) + \gamma d_{ij\prime}}} \qquad [12.2]$$

Gravity & Logit Model Equivalence (2)

Assume a logit destination choice model given by:

$$P_j^{ip} = \frac{e^{ln\left(X_j^p\right) + \gamma d_{ij}}}{\sum_{j' \in L^{ip}} e^{ln\left(X_{j'}^p\right) + \gamma d_{ij'}}} \qquad [11]$$

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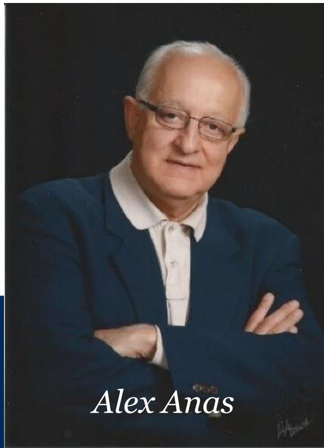
Gravity & logit are one & the same! (Anas, 1983).



Accessibility & Travel Modes (1)

If the probability of using mode m for a trip from i to j for purpose p from a set of feasible mode modes M_j^{ip} is given by:

$$P_{jm}^{ip} = \frac{e^{V_{jm}^{ip}}}{\sum_{m' \epsilon M_j^{ip}} e^{V_{jm'}^{ip}}} \qquad [13]$$

Then, the random utility theory definition of the modal accessibility for zone j is:

$$A_j^{ip} = ln\left(\sum_{m \epsilon M_j^{ip}} e^{V_{jm}^{ip}}\right) \qquad [14]$$

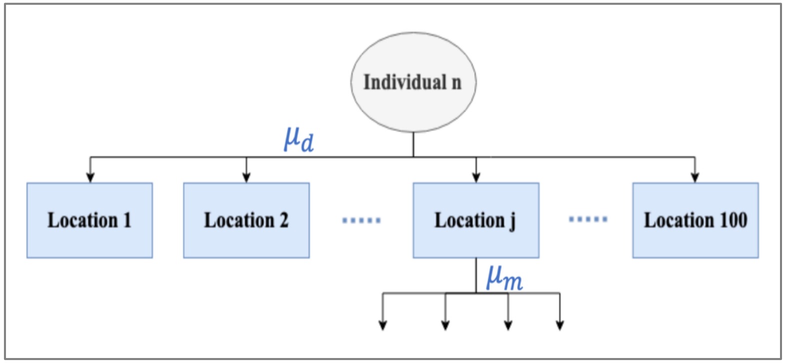
Accessibility & Travel Modes (2)

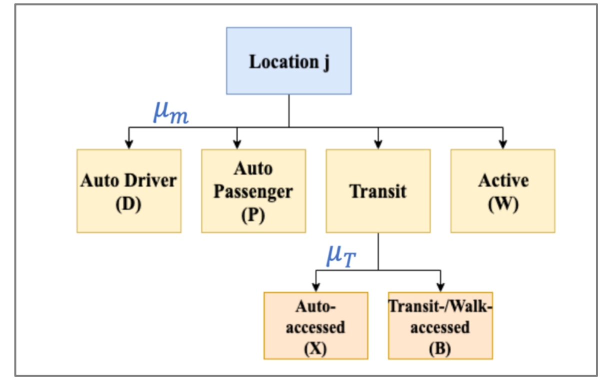
A multi-modal location accessibility measure that extends the single mode MNL-based location accessibility measure can then be constructed by adopting a *nested logit* model of the joint choice of location and mode (Ben-Akiva & Lerman, 1985; Train, 2009). The final result is a location choice model that takes the form:

$$P_j^{ip} = \frac{e^{\tilde{V}_j + \varphi A_j^{ip}}}{\sum_{j' \in L^{ip}} e^{\tilde{V}_{j'} + \varphi A_{j'}^{ip}}} \qquad [15]$$

where, $\tilde{V}_j$ is the systematic utility of location j, excluding travel-related utility (which is captured in the mode choice model logsum modal accessibility term A_j^{ip}) and φ is a "scale parameter" that must lie between zero and one in value for a properly specified model. The multi-modal location accessibility associated with this model is then:

$$A^{ip} = \sum_{j \in L^{ip}} e^{\tilde{V}_j + \varphi A_j^{ip}} \qquad [16]$$





E.g., Xi (2019) Nested Logit Work Location & Mode Choice Model, 2016 GTHA

Work Location Choice Model Parameters (All Occupation Groups, Full-(F) & Part-time (P))

Group		ρ^2	Variable	Value	Std err	Robust Std. Err.	t-test w.r.t. 0	Robust t-test w.r.t. 0	t-test w.r.t. 1	Robust t-test w.r.t. 1
G	F	0.213	ACCESS	**0.818**	0.016	0.0176	51.12	46.48	-11.36	-10.33
			LNEMP	**0.835**	0.0101	0.0109	83.11	76.89	-	-
			IFPD1	**0.386**	0.0288	0.0276	13.41	13.98	-	-
	P	0.198	ACCESS	**0.992**	0.0546	0.0584	18.16	16.98	-0.14	-0.13
			LNEMP	**0.467**	0.0468	0.0567	9.97	8.23	-	-
			IFPD1	**0.665**	0.124	0.123	5.37	5.4	-	-
M	F	0.031	ACCESS	**0.875**	0.0174	0.0177	50.17	49.54	-7.18	-7.06
			LNEMP	**0.260**	0.0096	0.0127	27.06	20.5	-	-
			IFPD1	**0.171**	0.0635	0.0629	2.69	2.72	-	-
	P	0.141	ACCESS	**0.995**	0.0759	0.0788	13.11	12.63	-0.06	-0.06
			LNEMP	**0.396**	0.069	0.0679	5.74	5.83	-	-
			IFPD1	-	-	-	-	-	-	-
P	F	0.203	ACCESS	**0.985**	0.0063	0.0065	156.86	151.62	-2.35	-2.27
			LNEMP	**0.886**	0.0054	0.0058	163.86	153.1	-	-
			IFPD1	**0.383**	0.0174	0.0182	22	21.06	-	-
	P	0.115	ACCESS	**0.975**	0.0275	0.0308	35.39	31.69	-0.91	-0.81
			LNEMP	**0.579**	0.026	0.0347	22.3	16.7	-	-
			IFPD1	**0.702**	0.0902	0.0973	7.78	7.21	-	-
S	F	0.122	ACCESS	**0.875**	0.0262	0.033	33.43	26.52	-4.77	-3.79
			LNEMP	**0.827**	0.0116	0.0129	71.33	64.01	-	-
			IFPD1	**0.306**	0.0339	0.0328	9.04	9.34	-	-
	P	0.188	ACCESS	**0.975**	0.0503	0.0629	19.39	15.5	-0.50	-0.40
			LNEMP	**0.570**	0.0267	0.0304	21.36	18.76	-	-
			IFPD1	**0.293**	0.0931	0.09	3.15	3.26	-	-

Work Mode Choice Model Parameters (Sales & Service Occupation group)

Name	Value	Std err	t-test	p-value	Robust Std. err.	Robust t-test	p-value
ASC_P	-1.087	0.15	-7.26	0	0.152	-7.14	0
ASC_X	-0.715	0.262	-2.73	0.01	0.271	-2.64	0.01
ASC_B	-0.037	0.102	-0.36	0.72	0.105	-0.35	0.73
ASC_W	1.531	0.21	7.3	0	0.252	6.08	0
COSTINC	-2.838	0.557	-5.1	0	0.542	-5.24	0
IVTT	-0.012	0.0017	-7.33	0	0.0016	-7.8	0
OVTT	-0.029	0.00341	-8.64	0	0.0035	-8.41	0
WALKDIST	-0.890	0.0742	-11.99	0	0.093	-9.57	0
AGE1624_B	1.428	0.121	11.84	0	0.114	12.58	0
AGE1624_W	1.464	0.227	6.45	0	0.224	6.52	0
AGE2564_P	-1.301	0.137	-9.5	0	0.138	-9.4	0
AGE2564_X	-0.964	0.161	-6.01	0	0.167	-5.78	0
AGE65_P	-1.777	0.247	-7.21	0	0.251	-7.07	0
AGE65_X	-1.937	0.315	-6.15	0	0.344	-5.63	0
AGE65_B	-0.572	0.175	-3.27	0	0.165	-3.47	0
TWOVEH_D	1.746	0.0796	21.92	0	0.0764	22.85	0
TWOVEH_P	0.641	0.105	6.08	0	0.106	6.07	0
TWOVEH_X	1.122	0.173	6.49	0	0.174	6.46	0
NOVEH_B	2.023	0.195	10.37	0	0.196	10.31	0
NOVEH_W	1.883	0.25	7.54	0	0.263	7.15	0
IFADJ_B	-0.925	0.177	-5.23	0	0.191	-4.86	0

Note: rho-squared is 0.539, μ(T) is 1.570

Issues in Using Accessibility Measures

- Lack of understanding of accessibility concepts among politicians, the public & non-modellers in general.
- Technical limitations within planning agencies.
- Concerns about: computational complexity, standardized software availability, data, etc.
- Ordinal & subjective nature of accessibility.
- Economic valuation of accessibility.

Ordinal & Subjective Accessibility (1)

- Utilities are ordinal, not cardinal (interval, not ratio) numbers and so no absolute value of utility – and, hence, utility-based accessibilities – exists.
- This problem is compounded in disaggregate measures, wherein utility parameters vary across persons.
- As a result, comparing accessibilities across persons, and establishing valuations is challenging.

Ordinal & Subjective Accessibility (2)

- Simple logit models actually have unidentifiable "scale" (μ) and "shift" (C) parameters that are normally embedded in the utility function utility parameter estimates β.
- Explicitly showing these, a typical logit logsum accessibility term becomes:

$$A^{ip} = ln\left(\sum_{j \in L^{ip}} e^{(\boldsymbol{\beta} \boldsymbol{Z}_j + C)/\mu}\right) \qquad [17]$$

- Accessibilities computed for different groups (e.g., different classes of workers), activities (employment access vs. shopping access), different cities, etc. will not be numerically comparable since the (C,μ) parameters will be different in unknown ways.

Ordinal & Subjective Accessibility (3)

To convert scaled accessibilities, A^{ip}, into de-scaled accessibilities, $\widetilde{A^{ip}}$, that can be compared across groups, etc. define the following parameter, α:

$$\alpha_{p,z} = \frac{A^{ip}(\Delta z) - A^{ip}(Base)}{\Delta z \sum_j \sum_m P^{ip}_{jm} z^i_{jm}} \qquad [18]$$

Where:

$A^{ip}(Base)$ = Base accessibility for zone i and activity/group p

Δz = A fixed, marginal change in an explanatory variable z, typically travel cost or travel time that is applied to origin-destination (O-D) travel times

$A^{ip}(\Delta z)$ = New accessibility based on Δz being added to O-D travel times

P^{ip}_{jm} = Probability that a person in zone i will travel to zone j by mode m, computed by whatever location mode choice model is associated with the accessibility measure being used

z^i_{jm} = Travel time by mode m from i to j

Ordinal & Subjective Accessibility (4)

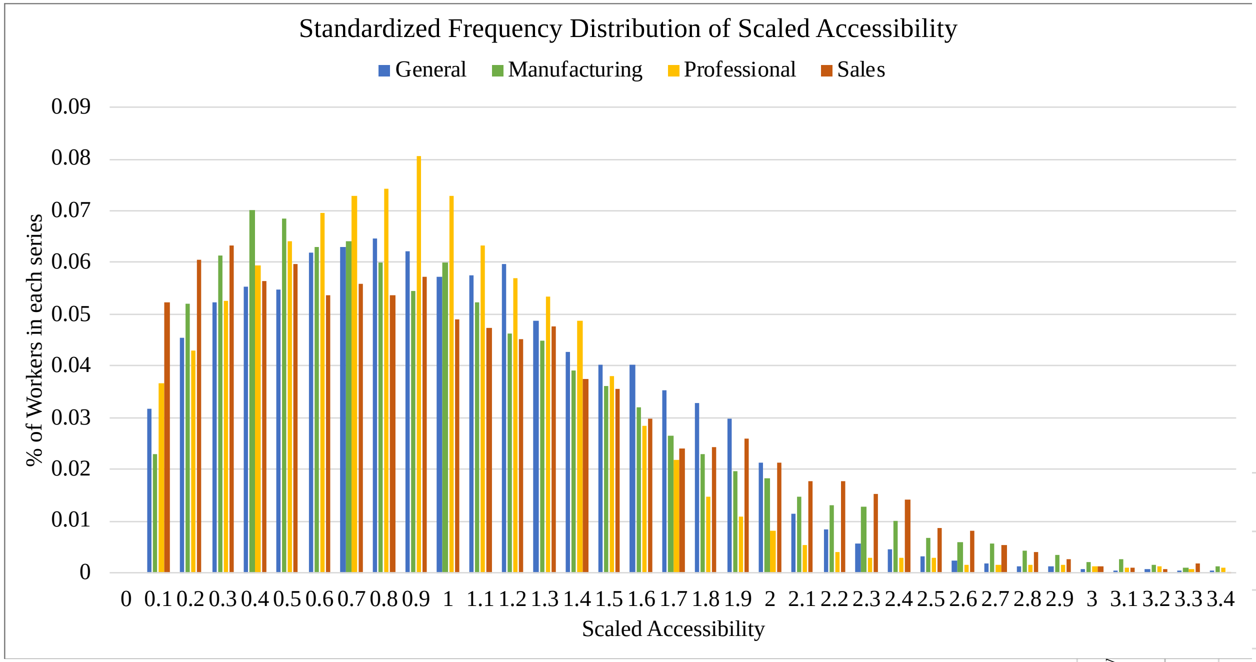
$$\alpha_{p,z} = \frac{A^{ip}(\Delta z) - A^{ip}(Base)}{\Delta z \sum_j \sum_m P_{jm}^{ip} z_{jm}^{i}} \qquad [19]$$

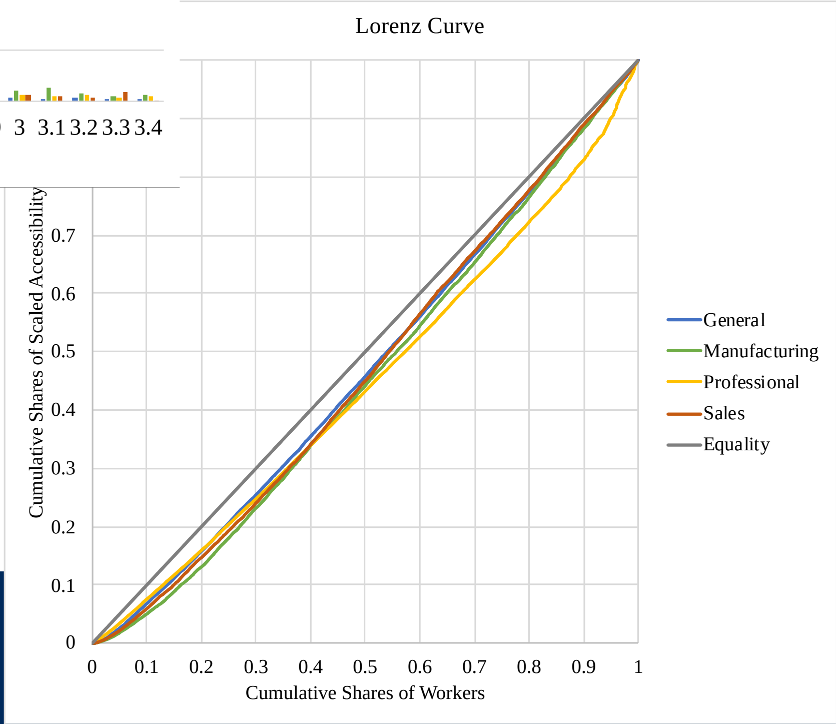
The numerator of Equation [19] eliminates the unidentified shift effect in the model. α can be interpreted as an empirical approximation of the marginal accessibility with respect to a change in z, across all locations in the system (Dong, et al., 2006). Its units are utils^p per monetary unit (dollars, euros, etc.) or minute, depending on what variable z represents and the category p. $\widetilde{A^{ip}}$ can then be defined as:

$$\widetilde{A_z^{ip}} = \frac{A^{ip}}{\alpha_{p,z}} \qquad [20]$$

$\widetilde{A_z^{ip}}$ is expressed in the same units as z, either monetary units or minutes, as the case may be, and so may be directly compared to other "de-scaled" accessibilities (Xi, 2019).

Ordinal & Subjective Accessibility (5): An Example





2016 GTHA Workers Scaled Accessibility to Employment Locations

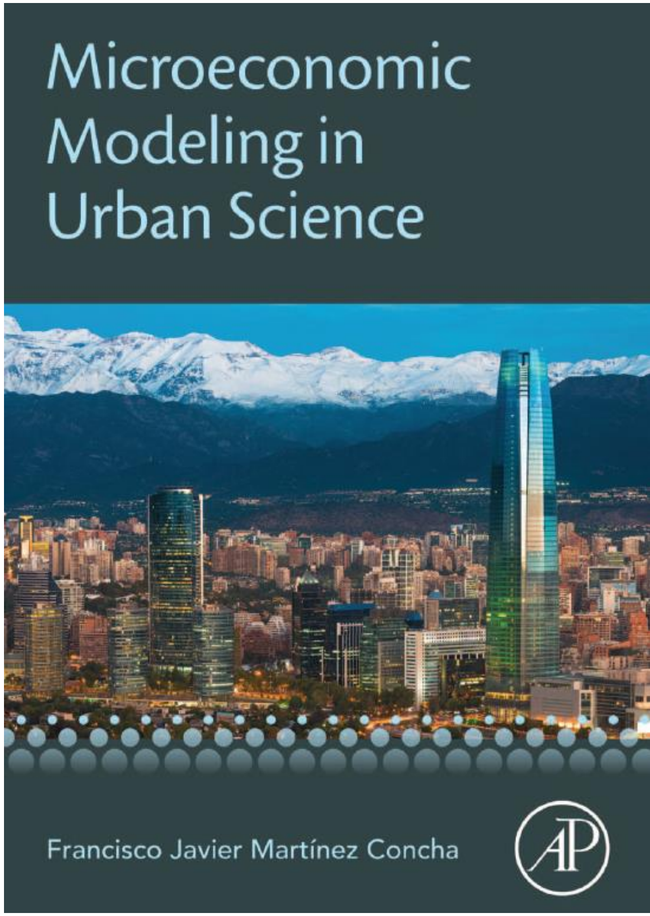
(Xi, 2019)

Issues & Options in Valuing Accessibility (1)

- Simplistic location utility functions (e.g., $e^{ln\left(X_j^p\right) + \gamma d_{ij}}$) treat all opportunities at each location as being equally attractive.
- Improved utility functions with actual "attractiveness" variables would:
  - Differentiate among competing locations, other than on a basis of size.
  - Provide a "hedonic" utility that would provide a better basis for valuation.
- E.g., surely income would be a useful attribute to include in employment accessibility measures?

Issues & Options in Valuing Accessibility (2)

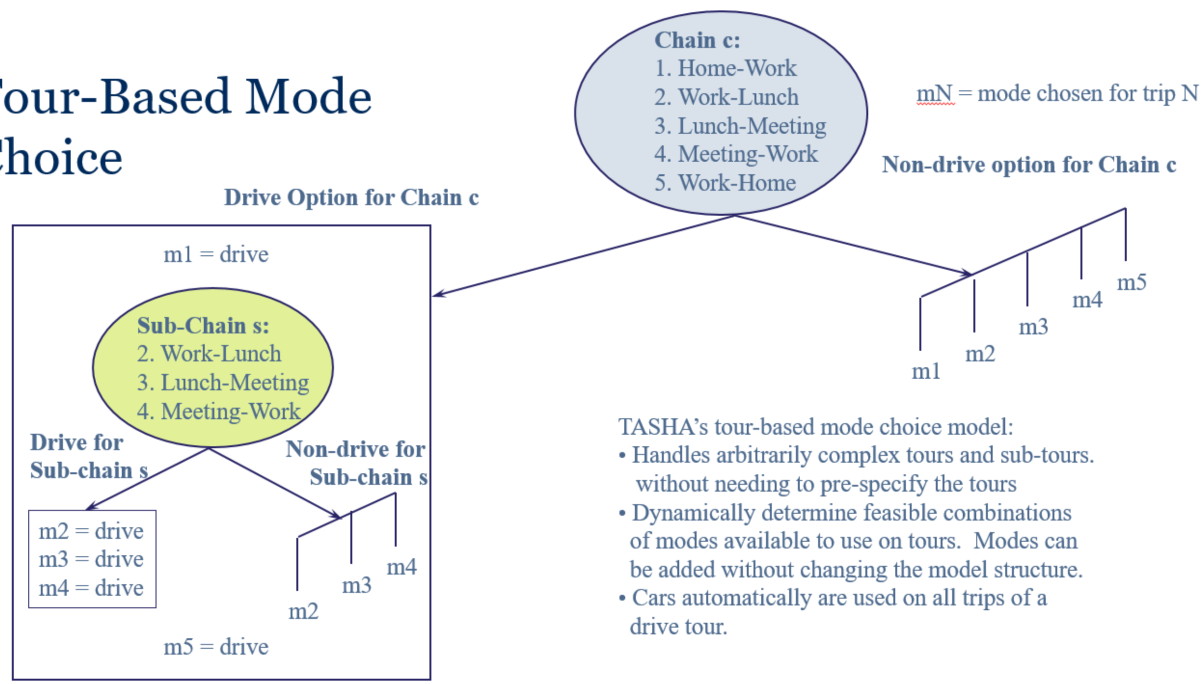
- Residential location choice models within integrated land use – transportation models are one approach to establishing a "market valuation" of accessibility.
- Accessibilities can enter the residential location utility function. Multiple accessibilities can be included (access to jobs, schools, shopping, etc.).
- The model provides parametric assessment of how accessibilities are valued relative to other residential location factors (dwelling unit & neighbourhood attributes, housing cost, etc.).
- Providing that the utility function can be inverted, hedonic prices for accessibility can be derived (Martinez, 2018).



Issues & Options in Valuing Accessibility (3)

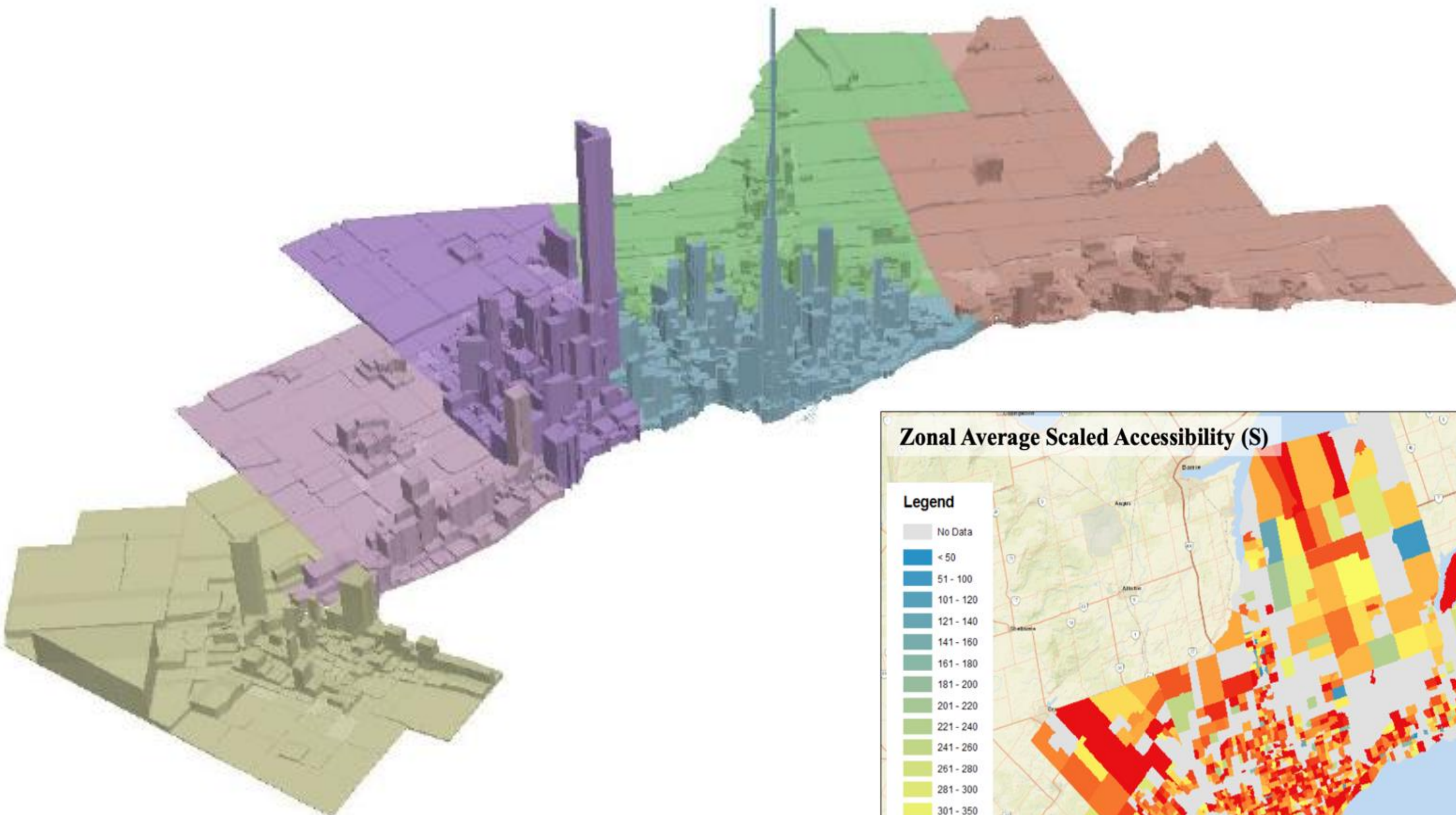
- Activity-based models (Miller, 2018a) provide a more comprehensive view of travel behaviour:
  - They can provide a holistic view of access to all activity types, by all modes by all household members over the course of an entire day.
  - Not yet used in operational applications (but could be).

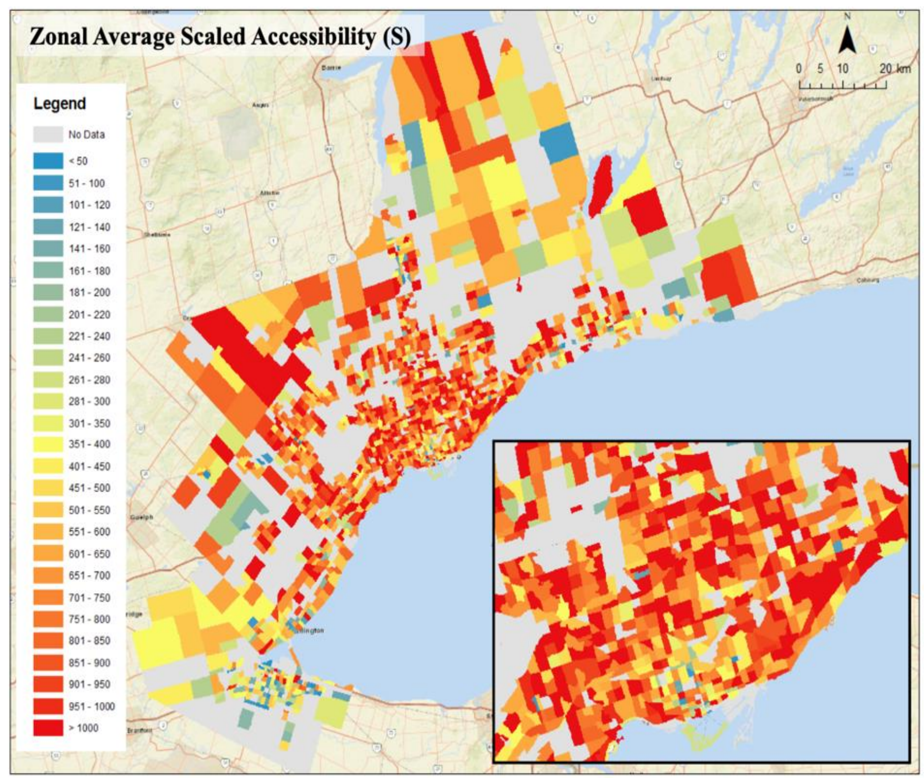
Tour-Based Mode Choice



TASHA's tour-based mode choice model:
- Handles arbitrarily complex tours and sub-tours. without needing to pre-specify the tours
- Dynamically determine feasible combinations of modes available to use on tours. Modes can be added without changing the model structure.
- Cars automatically are used on all trips of a drive tour.

Thank you! Questions?





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