International Transport Forum

Measuring Accessibility
Methods and Issues

182
Roundtable

Discussion Paper

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The International Transport Forum

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Introduction

Transport accessibility is defined as the *potential* for participating in activities (or, equivalently, interacting with people) that are distributed over space. Intuitively, the more opportunities that are available to a person to participate in a given type of activity, the more attractive these opportunities are for engagement, and the easier it is to travel to these activity locations, the higher one's accessibility (Páez, Scott and Morency, 2012). For example, if a person lives near many stores selling high quality goods at reasonable prices, and if it is easy and convenient to travel to these stores, then that person has a high level of accessibility for shopping relative to someone who lives in a remote area that is far from stores and/or whose only local stores are few and small, selling goods of poor quality at high prices.

Accessibility can be differentiated from *connectivity* on the one hand, and *mobility* on the other. Connectivity deals with the extent that one point (node) in a network is connected to other points (nodes) in the network. Connectedness can be characterised by various graph theoretic measures, such as coverage, directness and connectivity (Derrible, 2009). Connectivity concerns are important in transport network design, especially transit networks. In general, high levels of connectivity are desirable so as to provide redundancy and hence network resiliency (if one link is temporarily removed from the network, alternative paths still exist to maintain point-to-point connectivity), to spread flows across multiple paths within congested networks (thereby reducing overall congestion-related delays in the system), and to enhance accessibility: good connectivity is a necessary (but not sufficient) condition for providing good levels of accessibility.

Mobility involves the actual movement of people (and goods) from point to point within the transport system. It thus represents the actualisation of accessibility (i.e. the potential to travel and interact) in terms of actual trips and interactions. The vast majority of transport planning, modelling, policy analysis and decision making, deal with questions of mobility: minimising congestion and travel times, influencing travel mode choices, reducing transport greenhouse gas emissions, traffic accidents and other undesirable externalities of travel, etc. Measures of system benefits in most planning studies are mobility based: trip-maker travel time reductions, for example, are virtually universally used as a primary measure of the benefits accruing from an improvement in transport system infrastructure or other changes in the system.

Despite this focus on mobility, it is highly arguable that the "primary role of a transport system is to provide people and businesses with access to other people and businesses so that they can physically engage in spatially and temporally distributed activities of all kinds, and so that they can physically exchange information, goods and services" (Miller, 2018b). If this proposition is correct – i.e. that a major objective of transport is to provide high levels of accessibility, then it would be logical that accessibility measures should play a major role in transport analysis and planning: how can the impacts of policies on improving accessibility be assessed, if accessibility is not measured? This concern for using accessibility measures in transport planning and evaluation is particularly important from the perspective of equity analyses. Poor accessibility is a primary constraint for transport-disadvantaged groups within society. Such groups often display low mobility levels, not because they do not wish nor need to travel, but because their poor accessibility makes travel difficult. Further, and more fundamental, it is not the lack of travel that is the problem *per se*, but rather it is the lack of access to (ability to interact with) employment, schools, health care, healthy food, etc. that is the root problem. That is, it is not a lack of mobility, it is a lack of access.

While the logic for accessibility-based planning seems clear, accessibility measures typically play little or no explicit role in many planning exercises. Many reasons contribute to this state of affairs. This paper focuses on issues concerning the measures themselves: their definitions, theoretical foundations and practical problems in their usage in operational planning contexts. The next sub-section elaborates the loose definition of accessibility presented above in terms of a set of axioms that help formalise the definition of accessibility and that provide criteria that should be met in the specification of any operational accessibility measure. The second section of the paper then presents a typology of accessibility measures in general use, with an emphasis on their theoretical foundations and similarities and differences. Building on this theoretical foundation, the paper's third section discusses several issues associated with constructing and using accessibility measures in planning practice. The fourth and final section summarises the key messages of the paper and proposes a few next steps in improving accessibility measures and advancing their usage in practice.

Accessibility axioms and assumptions

General agreement exists among academics and practitioners concerning a set of axiomatic properties of accessibility that any valid accessibility measure should satisfy. These are as follows (Miller, 2018b):

1. Accessibility varies from one point in space to another. "Point" in this context may mean a particular X-Y geocode, a building or parcel, or a zone centroid (traffic zone, census tract, etc.). People living in denser, central urban areas, for example, are likely to have higher accessibility levels than people who live in low-density rural areas, since they are closer to more activity opportunities.
2. Accessibility is activity (trip purpose) specific. A given location may have good access to stores for shopping, but poor access to employment opportunities, for example.
3. Accessibility intrinsically combines a measure of the ease/difficulty involved in travelling to or interacting with different points in space (typically referred to as the *disutility*, *generalised cost* or *impedance* of travel) with a measure of the *attractiveness* and/or *magnitude* of opportunities (i.e. the desirability and/or number of activity opportunities) at different spatial locations.
4. Specifically, the measurement of accessibility involves the integration or summation of opportunities over space, weighted by the ease of interaction; i.e. opportunities that are closer/easier to access generally will be weighted higher than those that are further away or more difficult to reach in terms of determining the overall accessibility level.

Developing a computable accessibility measure that is consistent with these four axioms requires decisions concerning four additional assumptions (which, in turn, will spawn the need for further, more detailed assumptions) in order to operationalise several of the terms used in these axioms. These are the definition of:

1. travel disutility or impedance
2. location attractiveness
3. the role of individual tastes, preferences and constraints in determining both travel impedance and location attractiveness
4. the set of locations to be included in a given accessibility calculation.

Determining these four assumptions lies at the heart of developing a specific, operational accessibility measure. The options and issues for doing so are addressed in detail in the next two sections of this paper. In the remainder of this section, however, a few preliminary remarks concerning each of these assumptions are presented to lay the foundation for the more detailed, technical discussions in the subsequent sections.

Travel impedance

The simplest accessibility measures use distance as the impedance metric. Depending on the application, this distance might be computed as simply the straight-line (Euclidean) distance between two points, the right-angled (Manhattan) distance travelled along a rectangular grid network from point to point, or the network distance, typically computed as the shortest-path from point to point through the actual network.

For walk-based accessibilities, distance is generally a suitable impedance metric, since it correlates very strongly with travel time and it also correlates equally strongly with the amount of personal effort required to make the trip. For other modes of travel, however, travel time is a much better measure of the effort or cost involved in making a trip by car, transit or even bicycle. Whether a trip is 5 km long or 10 km long matters much less than the amount of time expended to make the trip; if both these trips take 15 minutes to complete, then presumably the accessibility to both locations (assuming that they are equally attractive) should be the same. Further, travel time is a policy-sensitive variable (i.e. improvements in transport system performance will reduce travel times, thereby improving accessibility, and vice versa) whereas distance usually is not. Thus, travel-time based measures are generally much more useful variables to include in planning analysis and evaluation, since they permit differences in accessibility impacts of different policies and alternatives to be computed.

The introduction of travel time as the preferred impedance metric adds a new dimension of complexity to the definition of accessibility, since it means that accessibility for any activity type varies by travel mode. For a given spatial distribution of activities, faster travel modes (such as, typically, automobiles) will *ceteris paribus*, provide higher levels of accessibility than slower modes (such as, typically, transit and walking). Further, since modal travel times vary by time of day (primarily due to diurnal variations in roadway congestion levels and transit service levels), this means that accessibility for a given activity by a given mode can vary by time of day as well.

As is discussed further in the next section, more generalised utility-based measures of impedance are also possible that incorporate other trip attributes, such as travel cost, in addition to travel time. These additional attributes will also vary by mode, and, often, time of day (e.g. parking availability and cost for the auto mode).

Location attractiveness

The simplest, and most common, assumption for defining a location's attractiveness is to use a size variable of some sort: the number of jobs in a given zone for employment accessibility, the number of stores (or retail floor space or retail employees) in a zone for shopping accessibility, etc. More complex representations involving other attributes of an activity location affecting its attractiveness for potential interaction are conceivable (such as the quality and price of goods for shopping accessibility). But, in practice, such more detailed characterisations of attractiveness are rarely used, presumably due to data limitations and the additional analytical complexity that would be introduced.

Note that the attractiveness and/or availability of a given activity location might also vary over the course of the day due to factors such as store opening/closing hours and congestion levels within the facility (e.g. a popular restaurant may be fully booked and so not available without an advance reservation).

Person-level heterogeneity in accessibility

Both the (dis)utility of travel to a given location and the assessment of the location's attractiveness as a potential destination can be expected to vary subjectively from person to person due to varying personal tastes and preferences, as well as variations in personal resources and constraints. High-income people generally will have greater access to a much wider range of goods, services and activities than low-income people due to their greater buying power. Persons with access to cars will have wider activity ranges than those who do not own cars and/or are unable to drive. Thus, two people located at the same point in space at the same point in time might have different accessibility levels due to a combination of differences in both their personal constraints and preferences.

The recognition (and measurement) of inter-personal variations in accessibility may often be critical in policy analysis, especially for equity considerations and for targeting accessibility-disadvantaged groups for special consideration. It also, however, clearly introduces significant additional complexity into the accessibility calculations that may or may not be supportable in terms of the associated additional data, computational and technical requirements. Many accessibility measures side-step this complexity by working at the aggregate level, typically using traffic zones or census tracts as the unit of analysis. This effectively treats all persons living within a zone as being homogeneous in terms of their preferences and constraints.

Location choice set

Fourth, the question of what locations are to be included in a given accessibility calculation is inherent in all accessibility measures. If one is computing accessibility to employment opportunities in the City of Toronto, presumably jobs located in Montreal (over 500 km away) are irrelevant, but what about jobs in Hamilton (70 km away)? As discussed in the next two sections, different accessibility measures address this question in different ways, and different modellers often make different assumptions even within the same type of measure.

Accessibility and travel demand

There is clearly a logical connection between the concept of accessibility and travel demand. People reveal their preferences for different activity locations and modes of travel through their travel destination and mode choices. If accessibility is the measure of how people value the opportunities available to them within their action space, then this measure should be consistent with how these opportunities are evaluated when people decide how to actually engage in an activity at a given location within this set of opportunities. Thus, any operational measure of accessibility should be consistent with or derived from people's travel demand preferences and constraints. This assertion is demonstrated in the next section, in which it is seen that operational accessibility measures do, indeed, emerge out of more fundamental assumptions concerning travel demand decision-making.

A typology of accessibility measures

While many variations of accessibility measures exist, they can largely be grouped into four primary categories (Handy and Niemeier, 1997; Kwan, 1998):

1. Distance (or travel time by mode) to the nearest subway station, freeway interchange, shopping centre, medical centre, etc.
2. Cumulative opportunities reachable within a maximum access distance or travel time threshold (isochrone method)
3. Gravity, or entropy methods, "Hansen's measure"
4. Random utility-based measures.

Each of these measure types are discussed in detail in the following sub-sections. To keep the discussion relatively simple, the case of measuring accessibility from an origin zone to a set of opportunities is consistently considered. Extension of the measures discussed to computing accessibilities for individual persons is generally mathematically straightforward (in cases in which the measure definition permits this extension.

Distance to nearest location

Mathematically, this measure can be expressed as:

$$A^{ip} = \underset{j \in L^p}{MIN}(d_{ij}) \qquad [1]$$

Where:

A^{ip} = Accessibility of zone i to location of type p

L^p = Set of locations of type p

d_{ij} = Distance (or travel time for a given mode) from i to location j in set L^p

This is a very restricted accessibility measure relative to the definitions presented in the previous section in that it:

- Does not consider the size/attractiveness of the closest location, thereby implicitly treating all locations as being equally attractive.
- Does not consider the cumulative effect of multiple accessible locations (e.g. is a zone that is within 1.1 km of two subway stations inferior to one that is within 1.0 km of a single station?).

This measure is consistent with an extremely simple location model in which the nearest location is always chosen with probability 1.0. That is:

$$P_j^{ip} = 1 \; if \; d_{ij} = \underset{j' \in L^p}{MIN} d_{ij'} \; ;= \; 0 \; otherwise \qquad [2]$$

Where, P_j^{ip} is the probability of choosing location j for purpose p given that one is located in zone i. This is, except in special cases, not a realistic choice model, whether one is talking about a subway station or freeway interchange (which will only be used if it is on a best path for a given trip to the actual trip destination) or an activity location (for which there generally will be many competing locations of varying attractiveness). Thus, this measure is better suited as an explanatory variable in mode or residential location choice models rather than as a stand-alone accessibility measure. Given this, it is not considered further in this paper.

Isochrone, or cumulative count measures

Probably the most common accessibility measure used in practice is the isochrone or cumulative count approach. It is defined by the following equation:

$$A^{ip} = \sum_{j \in L^p_{D|i}} X^p_j \qquad [3]$$

Where:

$L^p_{D|i}$ = Set of locations of type p that are within a maximum distance (or travel time) D of zone i

X^p_j = Size of activity type p (number of jobs, stores, etc.) at location j

This accessibility measure is consistent with a location choice model of the form:

$$P^{ip}_j = \frac{X^p_j}{\sum_{j' \in L^p_{D|i}} X^p_{j'}} \;\; if\; j \in\; L^p_{D|i} \;\; ; =\; 0\; otherwise \qquad [4]$$

The primary advantages of this measure are that it is simple to compute (especially with ubiquitous access to Geographic Information System (GIS) software and databases) and intuitive to understand. Given this, it certainly a useful measure in a number of applications. The measure, however, has several serious theoretical and methodological issues that arguably significantly limit its general use. These include the following.

First, the measure assumes that people are indifferent among travel distances/times to competing locations, as long as they all lie within the threshold D. While it may be arguable that people are relatively indifferent to small differences between short distances/times (e.g. whether one needs to travel 20 or 22 minutes to competing locations), the principle that people would rather spend less time/effort travelling than more is fundamental to travel behaviour theory. As an extreme but still useful edge case, consider two cases: one in which a given set of locations are located exactly 30 minutes from a given zone (for a case in which D equals 30 minutes), and one in which these same locations are located 5-10 minutes away from the zone. Surely the latter case delivers a higher accessibility level than the first.

Second, and similarly, the measure assumes that all locations located beyond the threshold (even by an infinitesimal amount ε) are irrelevant to location i's accessibility and that the probability of visiting these locations is zero. This is clearly unsupportable within travel behaviour theory. Again, consider the edge case (again with D = 30 minutes) in which one scenario consists of a set of locations all located at 29.9 minutes from zone i and one in which this same set of locations is situated 30.1 minutes away. Does zone i's accessibility really drop to zero in the second case (or is much different than for case one)? The answer is, obviously, no.

Consider the case in which all locations within a distance threshold are all of the same size, X. In this case, the probability of choosing a location is simply 1/N for any location within the threshold (where N is the number of locations) and zero otherwise. A much more behaviourally plausible (and empirically verifiable) model is one in which the probability of a location's choice declines with distance/travel time. This decline may be slight to begin with (relative indifference among locations with similar, short distances/time) and then decline relatively precipitously in the neighbourhood of some threshold (at which point the travel impedance is becoming increasingly and discernibly onerous), and then beyond some point the choice probabilities become vanishingly small. Clearly, an accessibility measure consistent with this behaviour must be preferred to the unrealistic isochrone behavioural assumption.

Third, the definition of the threshold D is arbitrary and not derivable from either theory or empirical estimation. It must be asserted. No standard threshold exists in the literature or in practice. Thirty, forty, and forty-five minute thresholds are probably the most common values used, but these are always chosen in an *ad hoc* basis, usually based on loose assertions that most trips occur within the given range, or that people are indifferent among travel times within the given range. Such assertions have no strong basis within travel behaviour theory. Further, and much more problematic, it can be shown that the relative ranking of zonal accessibilities can change in arbitrary ways depending on the threshold assumed (Xi, Miller and Saxe, 2018). That is, a 30-minute threshold can generate a different relative ordering of zonal accessibilities relative to that generated by a 40-minute or a 45-minute threshold. This is very concerning since (as is discussed much further in the third section), current accessibility measures (including isochrone-based measures) can only provide relative orderings of accessibility (zone i has better accessibility than zone i'). If this rank ordering depends arbitrarily on an *ad hoc* threshold assumption, then the usefulness of the measure is seriously questionable.

Gravity, or entropy measures

A second very common accessibility measure is a wide variety of so-called *gravity* measures. These are commonly linked back to Hansen's seminal paper (Hansen, 1959). In their simplest (and most common) form they can be expressed as:

$$A^{ip} = \sum_{j \in L^{ip}} X_j^p f(d_{ij}) \qquad [5]$$

Where:

L^{ip} = Set of locations of type p in the choice set for zone i

$f(d_{ij})$ = Impedance function; $\partial f / \partial d_{ij} < 0$

In comparing equations [5] and [3] two key differences between the gravity and the isochrone approaches become evident. In the gravity approach the attractiveness of locations are weighted by the impedance function: locations which are closer are weighted more heavily than locations that are further away. The set of locations considered in the accessibility calculation is determined by the choice set, L^{ip}, rather than an arbitrary cut-off threshold.

As discussed below, the definition of this choice set is also problematic, but the flexibility provided by this approach is a useful step forward.

Thus, the isochrone measure is a special case of a gravity measure in which $f(d_{ij}) = 1$ and $L^{ip} = L_{D|i}^p$

Equation [5] is consistent with a location choice model of the form:

$$P_j^{ip} = \frac{X_j^p f(d_{ij})}{\sum_{j' \in L^{ip}} X_{j'}^p f(d_{ij'})} \qquad [6]$$

Equation [6] will generate a continuously declining choice probability with increasing distance/time and so represents a significant improvement over the isochrone approach from a behavioural point of view.

Equation [5] is conventionally referred to as a gravity measure because its associated location choice model (equation [6]) is one variant of the well-known gravity spatial interaction model. Gravity models have a very long history of usage in both geography and travel demand modelling. They get their name from their original derivation by analogy to Newton's Law of Gravity in that the spatial interaction between two points in space is assumed to be proportional to the size of the attraction location (X_j^p) and inversely related to the distance/time separating the two points ($f\left(d_{ij}\right)$).

Gravity models have been criticised from a number of perspectives, not least of which is that developing a model of human spatial interaction based on an analogy to Newtonian physics may be a rather dubious proposition. It can, however, be shown that equation [6] (and other, more complex gravity model variants) can be derived from Shannon's "information theory" (Shannon, 1948) as the least-biased (or, equivalently, most-likely) estimate of the spatial interaction probabilities (P_j^{ip}), given the known information concerning the system (the size and distance/time variables) (Wilson, 1967; Webber, 1977). When derived from information theory principles, these models are often referred to as *entropy* models since an entropy measure is used within the methodology to quantify the information contained within the model. The entropy approach to developing these models is critically important from both theoretical and practical perspectives. In terms of theory, it provides a sound statistical justification of the model and its functional form. In practical terms it also provides a methodology for the specification of both the attraction variables and the impedance functions, as well as for estimating impedance function parameters (Wilson, 1967).

Note that in both the isochrone and gravity/entropy cases, accessibility is defined as the denominator of the location choice model. While a plausible assumption, since this measure satisfies all the accessibility axioms and is a reasonably intuitive formulation, it is, nevertheless, somewhat *ad hoc* in nature. That is, that a suitable measure of accessibility is the impedance-weighted sum of the total opportunities located within a given set of feasible locations. Tying this definition a bit more closely to actual location choice, if we define $X_j^p f\left(d_{ij}\right)$ as the "accessibility" of zone j for zone i, then the probability that zone j is actually chosen for an interaction is the ratio of its individual accessibility to the total accessibility (the total set of options) available to zone i; again, a reasonable proposition.

Random utility-based measures

The final major type of accessibility measure considered in this paper is derived from random utility theory. Random utility theory is an extension of neo-classical microeconomic theory to address the modelling of discrete choices that accounts for the probabilistic nature of these choices from the modeller's perspective. The general problem is to predict a person's choice of one alternative from a feasible set of discrete choices. In travel demand models, classic examples of this problem include choice of travel mode and/or destination for a given trip. While the decision maker is assumed to be rational and hence will chose the alternative that they perceive to be the best, as an outside observer of the process, the modeller cannot say with certainty which alternative this will in fact be. Hence the modeller cannot say with certainty which alternative will be chosen, but can only, at best, try to assess the probability of each feasible alternative being chosen. The literature on random utility theory is vast and is not discussed in any detail in this paper. For a detailed discussion of the theory and its many applications, see, among others, Ben-Akvia and Lerman (1985), Ortuzar and Willumsen (2011) and Train (2009). By far the most common form of random utility model is the multinomial logit model (MNL), whose general form for the case of a destination choice model is given by:

$$P_j^{ip} = \frac{e^{V_j}}{\sum_{j' \in L^{ip}} e^{V_{j'}}} = \frac{e^{\boldsymbol{\beta Z_j}}}{\sum_{j' \in L^{ip}} e^{\boldsymbol{\beta Z}_{j'}}} \qquad [7]$$

Where:

$V_j = \boldsymbol{\beta Z}_j$ = The *systematic utility* of alternative j

$\boldsymbol{Z}_j$ = Vector of explanatory variables

$\boldsymbol{\beta}$ = (Row) vector of parameters

The actual perceived utility by a decision maker is:

$$U_j = V_j + \varepsilon_j \qquad [8]$$

where ε_j is the individual's idiosyncratic deviation in terms of how they perceive the utility of alternative j relative to the population average utility, V_j. The person chooses the alternative that generates the maximum perceived utility, U_j. This actual perceived maximum utility is unobservable, but, for the case of the MNL model, it can be shown (Ben-Akiva and Lerman, 1985) that the *expected maximum utility* (I^{ip}) associated with this choice is given by:

$$I^{ip} = E\big[MAX_j(U_j)\big] = ln\left(\sum_{j \in L^{ip}} e^{\boldsymbol{\beta Z}_j}\right) \qquad [9]$$

That is, it is the natural logarithm of the denominator of the logit choice model (commonly referred to by the term "logsum"). Further, it can also be shown that this expected maximum utility is the *consumer's surplus* for this choice. Thus it is a standard measure of economic benefit. Given this, Ben-Akiva and Lerman (1985) argue that it also provides a behaviourally and economically sound definition of accessibility: "accessibility for a given activity is the expected utility that would be derived from participation in this activity, which is also the consumer surplus associated with this participation". That is:

$$A^{ip} = ln\left(\sum_{j \in L^{ip}} e^{\boldsymbol{\beta Z}_j}\right) \qquad [10]$$

Gravity and logit model equivalency

In a seminal paper, Anas (1983) demonstrated that, if equivalently specified, gravity/entropy and random-utility-based MNL models are mathematically identical. To illustrate this, consider a simple specification of equation [7] in which the logit destination choice model is given by:

$$P_j^{ip} = \frac{e^{ln\left(X_j^p\right) + \gamma d_{ij}}}{\sum_{j' \in L^{ip}} e^{ln\left(X_{j'}^p\right) + \gamma d_{ij'}}} \qquad [11]$$

Now, in the gravity model in equation [6], define the impedance function to be $f\left(d_{ij}\right) = e^{\gamma d_{ij}}$, a very common (and theoretically well-justified) specification. Equation [6] then becomes:

$$P_j^{ip} = \frac{X_j^p e^{\gamma d_{ij}}}{\sum_{j' \in L^{ip}} X_{j'}^p e^{\gamma d i j'}} \qquad [12.1]$$

Noting that $X_j^p = e^{ln\left(X_j^p\right)}$, equation [12.1] can be rewritten as:

$$P_j^{ip} = \frac{e^{ln\left(X_j^p\right) + \gamma d_{ij}}}{\sum_{j' \in L^{ip}} e^{ln\left(X_{j'}^p\right) + \gamma d_{ij'}}} \qquad [12.2]$$

which is identical to the MNL model, equation [11]. This equivalency between the two model formulations is rarely remarked upon, possibly largely due to disciplinary silos. Geographers typically work within a gravity framework, often seemingly unaware of the connections of their models to random utility. Random utility modellers (often economists or engineers) typically tout their models as behaviourally superior to gravity models, often seemingly unaware of the gravity model's solid foundations in information theory. These different world views are often reinforced by the fact that gravity models are typically formulated at an aggregate (zone-based) level, with very simple specifications (such as in the examples presented above), while random utility models are typically applied at the disaggregate level of the individual trip-maker, with relatively extensive vectors of explanatory variables. But simple, aggregate MNL models are also used, and information theory provides the mechanism for generating complex impedance functions for disaggregate models which can be identical to random utility specifications.

Further, in addition to being mathematically equivalent in functional form, Anas (1983) also shows that, if estimated with the same base data, the parameter estimates for the two models are identical.

This mathematical equivalency between the two model formulations reinforces the theoretical defensibility of gravity/logit type models as the basis for modelling spatial interactions, and, by extension, accessibility. The information theory derivation demonstrates the statistical validity of the model. The random utility theory derivation ties the model to strong microeconomic theory of decision making.

Returning to accessibility, comparing the gravity and random utility accessibility measures, it can be seen that one is a monotonic transformation of the other; i.e. the random utility logsum measure is simply the natural logarithm of the gravity accessibility measure. Thus, both measures will generate the same relative ordering of accessibilities; i.e. if zone i has a larger gravity model accessibility score than zone i', then it will also have a larger logsum value. Whether the differences in the absolute numerical values between the two measures is meaningful will depend at least somewhat on the context in which these measures are being used. But given the logsum's direct tie to consumer surplus (and hence, economic benefit) it is arguable that it should be the preferred measure to use in most applications.

Accessibility and travel mode

In the discussions above, the accessibility measures have been defined in terms of access to a set of locations (or trip destinations), using a single distance or travel time (generically labelled herein as d_{ij}). As noted in the introductory section, however, travel time is mode-specific. Thus, when travel time is used to define zone-to-zone impedance (which is generally the preferred choice), then different accessibilities A^{ipm} should be computed for each mode m, using this mode's travel times, d_{ij}^{m}.

If a multi-modal accessibility measure is desired (i.e. one in which all available modes are considered simultaneously in a single measure), then two approaches are often used. The first is to define a "modal accessibility" for a single location based on a MNL mode choice model. That is, if the probability of using mode m for a trip from i to j for purpose p from a set of feasible mode modes M_j^{ip} is given by:

$$P_{jm}^{ip} = \frac{e^{V_{jm}^{ip}}}{\sum_{m' \epsilon M_j^{ip}} e^{V_{jm'}^{ip}}} \qquad [13]$$

Then, as discussed above, the random utility theory definition of the modal accessibility for zone j is:

$$A_j^{ip} = ln\left(\sum_{m\epsilon M_j^{ip}} e^{V_{jm}^{ip}}\right) \qquad [14]$$

A multi-modal location accessibility measure that extends the single-mode MNL-based location accessibility measure (equation [10], which is based on the single-mode MNL location choice model, equation [7]) can then be constructed by adopting a *nested logit* model of the joint choice of location and mode (Ben-Akiva and Lerman, 1985; Train, 2009). The final result is a location choice model that takes the form:

$$P_j^{ip} = \frac{e^{\widetilde{V}_j + \varphi A_j^{ip}}}{\sum_{j\prime\in L^{ip}} e^{\widetilde{V}_{j\prime} + \varphi A_{j\prime}^{ip}}} \qquad [15]$$

where $\widetilde{V}_j$ is the systematic utility of location j, excluding travel-related utility (which is captured in the mode choice model logsum modal accessibility term A_j^{ip}) and φ is a "scale parameter" that must lie between zero and one in value for a properly specified model. The multi-modal location accessibility associated with this model is then:

$$A^{ip} = \sum_{j\in L^{ip}} e^{\widetilde{V}_j + \varphi A_j^{ip}} \qquad [16]$$

Xi (2019) provides a recent example of the use of equations [13] to [16] to analyse worker accessibilities to employment opportunities in the Greater Toronto-Hamilton Area (GTHA).

Accounting for competition in accessibility measures

A final technical consideration in developing accessibility measures is the question of whether one needs to account for competition among agents in determining accessibility. A specific, important example of this is whether the competition among workers for jobs needs to be considered when computing workers' accessibility to employment, since each job can be filled by only one worker. If X_j^p in the models above is the number of jobs of type p located in zone j, the workers living in each zone i compete for these jobs, along with similar jobs in other employment zones. The models above ignore this competition, and so a worker living near a very large number of jobs (such as in the central area of most major cities), will be assigned a very high accessibility level. But if there are many other workers also living nearby and competing for these jobs, the effective accessibility may be much lower due to this competition. To account for this, one can impose a constraint on the location choice model that:

$$\sum_i P_j^{ip} = X_j^p \;\; \forall j \qquad [17]$$

That is, on average, the number of jobs in zone j will be exactly filled by workers living in residential zones located throughout the study area. Working through the information theory formulation of the location choice model eventually yields a new location accessibility measure that has the form (Xi, et al., 2020):

$$A^{ip} = \sum_{j\in L^{ip}} \frac{X_j^p f(d_{ij})}{B_j^p} \qquad [18.1]$$

$$B_j^p = \sum_i \frac{W_i^p f(d_{ij})}{A^{ip}} \qquad [18.2]$$

where W_i^p is the number of workers of type p living in zone i. B_j^p is a "balancing factor" that ensures that constraint [17] is satisfied in the associated location choice model. It also captures the competition among workers for the jobs located in zone j. In particular, the larger B_j^p is (representing high competition among workers for jobs in zone j), the smaller zone j's contribution to workers' accessibility.

Issues in using accessibility measures

As demonstrated in the previous section, typical accessibility measures are actually all special cases of a common abstract model of accessibility that is tied to location choice models that can be derived from either information theory or random utility theory. Specific implementations, however, will vary depending on application context, data availability, the analyst's disciplinary worldview and the extent to which the analyst adheres to (or is aware of) the theoretical underpinnings of the concept.

Many technical implementation issues exist with turning the abstract model into an operational tool, including: selection of explanatory variables, choice of level of (dis)aggregation, treatment of travel modes, parameter estimation/specification, and definition of the choice set (activity space) over which the measure is to be computed for each origin point/zone, among others. Numerous institution barriers also exist to more widespread usage of accessibility measures in transport planning. These include:

- lack of understanding of the concept of accessibility among politicians, the public and non-modellers in general
- technical limitations within planning agencies to support accessibility measurement development and applications
- concerns about computation complexity, standardised software availability and data availability, etc.

This section, however, focuses on two key issues with respect to the use of accessibility measures in planning analysis and decision making that are generally common across all measures. The first of these is the ordinal (and subjective) nature of accessibility which must be understood and dealt with in planning applications. The second related issue is how to attach value to accessibility in economic evaluation.

Ordinal and subjective accessibility

Although not necessarily apparent in the equations in the previous section, the utility of a travel mode or a destination is an ordinal, not a cardinal, variable. That is, there is no meaningful "zero value" for utility. Hence, while the difference ($V_1 – V_2$) between two utilities is meaningful, the ratio V_1/V_2 has no useful meaning. That is, if $V_1 = 10$ and $V_2 = 5$, then the difference between the two is five units, but V_1 is not "twice as good" as V_2 since the zero point in the calculation of the V's is arbitrary. To see this in a practical way, consider the logit model expressed in equation [7]. Any constant C of any magnitude and sign can be added to the utility of every alternative in the choice set and the choice probabilities for all alternatives will remain unchanged. It is only the differences between the alternatives' utilities that determine their choice probabilities, not the absolute values of the utilities. But if C is added to these utilities, the accessibility (equation [10]) becomes:

$$A^{ip} = ln\left(\sum_{j \in L^{ip}} e^{\boldsymbol{\beta z}_j + C}\right) \qquad [19]$$

which is numerically different from the equation [10] value. Thus, the absolute value of A^{ip} depends arbitrarily on the (unidentified) value of the "shift" parameter C and so has no universal, absolute interpretation. Further, also buried in the MNL equation is an unidentified scale or normalisation parameter, which also varies arbitrarily from one application to another.

The practical ramifications of these observations for using accessibility in planning analysis are at least threefold.

- Without further analysis (discussed below), accessibilities in one context (e.g. City one) and in another (e.g. City two), cannot be directly compared, since the unidentified scale and shift parameters are almost certainly different, rendering the absolute values of the accessibilities computed in the two cities inconsistent with one another; i.e. an accessibility value of A in City one may not mean the same thing as this same value of A computed in City two.
- Again without further manipulation, the units of A^{ip} are in "utils", which have no particular physical or intuitive meaning. If zone one has an accessibility of 300 and zone two has an accessibility of 400, zone two's accessibility is clearly relatively higher, but what does this difference of 100 utils mean in any practical sense?
- Going further, these observations mean that it is very difficult (if not impossible) to define meaningful thresholds for acceptable or good or poor accessibility. One can always construct a frequency distribution of, say, residential zone accessibilities to employment opportunities, and there will always be, for example, a bottom 10% of zones with the worst accessibility among this set of zones. But perhaps this region has excellent employment accessibility in general, and so even the worst zones still enjoy good accessibility relative to another urban region.

These issues are further compounded in disaggregate models in which the model parameters and/or explanatory variables often vary by person. Income, for example, may very reasonably be expected to affect mode and destination choices and so is often included in the model. This means, however, that the accessibility computed for a high-income person will be different than that computed for a low-income person, even if they find themselves in objectively identical circumstances (e.g. the same choice set, travel times and cost, etc.). Further, people are often categorised into relatively homogenous groups (e.g. workers might be grouped by occupation category) and separate models are developed for each group (resulting in different vectors of parameters – the β's – for the different groups). This means that the scale and shift parameters for each sub-model may well be different – again rendering direct comparisons of accessibilities for each group impossible.

If one is not interested in comparing accessibilities among different cities or across different groups, then these issues may be moot. The use of aggregate models also reduces (or at least obscures) this issue, although one then is faced with the potential for significant aggregation biases in the calculations that is often overlooked: if people are, indeed, heterogeneous in the preferences, constraints and needs, how meaningful is an aggregate, average accessibility?

If one wishes to compare accessibilities across groups or other contexts, the only ultimate solution is to de-scale the accessibilities so that they become numerically comparable. Dong et al. (2006), provide a method that can be adapted to convert scaled accessibilities, A^{ip}, into de-scaled accessibilities, $\widetilde{A^{ip}}$, that can be compared across groups, etc. This involves defining the following parameter, α:

$$\alpha_{p,z} = \frac{A^{ip}(\Delta z) - A^{ip}(Base)}{\Delta z \sum_j \sum_m P^{ip}_{jm} z^i_{jm}} \qquad [20]$$

Where:

$A^{ip}(Base)$ = Base accessibility for zone i and activity/group p

Δz = A fixed, marginal change in an explanatory variable z, typically travel cost or travel time that is applied to origin-destination (O-D) travel times

$A^{ip}(\Delta z)$ = New accessibility based on Δz being added to O-D travel times

P_{jm}^{ip} = Probability that a person in zone i will travel to zone j by mode m, computed by whatever location mode choice model is associated with the accessibility measure being used

z_{jm}^{i} = Travel time by mode m from i to j

The numerator of equation [20] eliminates the unidentified shift effect in the model, since it is cancelled out by the subtraction of the base accessibility from the updated value. It also, of course, is the net change in accessibility caused by a uniform change in z within the system. The denominator is the change in total travel cost/time due to the change in z. α can be interpreted as an empirical approximation of the marginal accessibility with respect to a change in z, across all locations in the system (Dong et al., 2006). Its units are utils^p per monetary unit (dollars, euros, etc.) or minute, depending on what variable z represents and the category p. $\widetilde{A^{ip}}$ can then be computed as:

$$\widetilde{A_z^{ip}} = \frac{A^{ip}}{\alpha_{p,z}} \qquad [21]$$

$\widetilde{A_z^{ip}}$ is expressed in the same units as z, either monetary units or minutes, as the case may be, and so may be directly compared to other de-scaled accessibilities (Xi, 2019).

Valuing accessibility

As discussed above, it is possible to convert accessibilities into monetary units. As also discussed above, if "logit logsum" accessibilities are used, then these can be interpreted as a consumer surplus measure. Hence such measures can be used directly in economic cost-benefit analyses if so desired. This is rarely done in current practice, perhaps due to the complexity of the calculations involved. Nevertheless, with current computing hardware and software capabilities and digital, GIS-based databases, there is no technical obstacle to doing so if a transport agency wishes to invest in developing this capability. In addition to this observation, three further issues are worth noting concerning the problem of valuing accessibility.

The first is the relative simplicity of typical location utility functions, in the most common case merely consisting of a "$ln(SIZE_j)$" term (such as $ln(Emp_j)$) plus a modal accessibility term for the given location. Such "utility" functions do not really capture the utility of participation in the given activity at a given location in any meaningful way. That is, they do not include any representation of the value of activity to the participant.

Indeed these formulations actually represent a very simplistic nested logit model in which it is assumed that only the size of the activity location matters, not the attributes of the activity *per se*. In the case of employment accessibility calculations, for example, the upper level is the choice of an employment zone j, with an implicit lower-level choice of a specific job in the zone. One can hypothesise the systematic utility of a given job k in zone j as being W_{jk} , where W_{jk} is a vector of attributes describing job k (salary, working environment quality, education/experience requirements, etc.) and $\boldsymbol{\mu}$ is a (row) vector of parameters. The upper-level location choice model systematic utility function could then be written as:

$$V_j = \theta ln\left(\sum_k e^{\mu W_{jk}}\right) + \varphi A_j^{ip} \qquad [22]$$

where θ is a scale parameter between zero and one in value. But if one assumes that all jobs in all zones are identical ($W_{jk} = W \; \forall \; j, k$) then for practical computational purposes, the logsum term in equation [22] can be replaced by $ln\left(Emp_j\right)$, the classic gravity model attraction term. Thus, virtually all operational employment accessibility measures implicitly assume that all jobs are equal in value to all workers.

This assumption clearly is not true. As illustrated above, it is technically possible (data permitting) to try to capture the heterogeneity in activity attributes within the accessibility measure. In the case of employment accessibility, good transport access to a large number of higher paying jobs (for, say, a given occupation group) would have a higher accessibility score than the same level of transport access to the same number of lower-paying jobs. Presumably, such a measure is a much better indicator of the economic benefit of accessibility than current, aggregate indicators.

The second observation is that another potential way of obtaining improved measurement of accessibility values is to incorporate accessibility terms in residential location choice models. Just as modal accessibility can be included as an explanatory variable in an employment location (or other activity location) choice model, so too can location accessibility terms be included in random utility residential location models, which also are usually formulated as MNL, or, possibly, nested logit models. The systematic utility function of such a model can be inverted to produce a "willingness-to-pay" or "hedonic price" function in which the dollar value attached to accessibility within a household's overall valuation of residential location alternatives can be computed (Martinez, 2018). Presumably, such values may provide an improved measure of accessibility value, since it is determined by considering how households trade off accessibility with other residential attributes (neighbourhood quality, dwelling unit attributes, etc.), as well as other significant budget items (automobiles, etc.). Further, multiple accessibilities (to jobs, to education, to shopping) can be included in these calculations, and so the relative values of these various accessibilities can be identified.

Third, and finally, note that all of the accessibility measures discussed to this point in this paper have been trip-based in nature; that is, they are based on an underlying model of travel behaviour involving making a single trip to a single location for a single purpose. But we actually organise our lives in terms of daily and weekly *activity schedules* and engage in daily travel organised in terms of *trip chains* or *tours*. State-of-the-art travel models recognise this reality and are *activity* or *tour*-based. That is, they model a person's and/or household's *daily activity pattern* and the travel that is generated in order to execute these patterns (Miller, 2018a).

This much more holistic approach to travel demand modelling provides an alternative framework for accessibility measurement. Rather than computing a number of separate measures by activity/trip purpose (and then possibly combining them within a residential location model to determine their relative values to a household), it is technically possible to compute a household's overall accessibility to the full range of activities available to them by calculating the total accessibility implied in the household's daily activity patterns (Dong et al., 2006). To the best of the author's knowledge, such an approach has not yet been operationally implemented. But it might provide a much more behaviourally sound, policy sensitive and perhaps even computationally attractive (since these accessibilities could be automatically computed within each run of the activity/travel model) than current methods.

Conclusion

This paper presents a technical discussion of the current state of practice in transport accessibility measurement. It demonstrates that while many measures exist in practice, to a large extent they are all special cases of a generic model of accessibility that conforms to a few fundamental axioms. Specific measures result from many implementation assumptions that are often driven by data availability, technical capability limitations on the part of the agency or analyst, and customary practice. Accessibility measurement is demonstrated to be fundamentally tied to travel behaviour (and the operational representation of this behaviour within travel demand models). These ties should be explicitly recognised (and exploited) if theoretically defensible and practically useful measures are to be constructed.

It is arguable that the accessibility state of practice significantly lags behind the achievable state of the art. This is unfortunate, since it translates into accessibility measurements generally not being used to their full potential (if they are used at all) to support practical planning analysis and decision making. It is also unnecessary, since modern computing technology (both hardware and software), associated big (and ubiquitous) digital databases and the technical modelling state of the art all have the capability of supporting much more detailed and useful measurements (Miller, 2019). While planning agencies are often reluctant to employ complex analytical methods, these can be very robustly encapsulated within software combined with very user-friendly interfaces that enable the planner/analyst to exploit the measurement tools without needing to be a modelling expert. Such capabilities need to be aggressively explored if advanced accessibility measures are to contribute significantly to future transport and urban policy debates going forward.

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The Accessibility Shift

This paper discusses the current state of transport accessibility measurement. It demonstrates that all commonly used measures are special cases of a generic accessibility model that conforms to a few fundamental axioms. The paper also shows that accessibility measures are fundamentally tied to travel behaviour. These ties should be explicitly recognised and exploited to construct theoretically defensible and practically useful measures.

All resources from the Roundtable on Accessibility and Transport Appraisal are available at:
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